[4323] – 101

Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2013
MATHEMATICS
MT-501 : Real Analysis – I (New Course)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) If l^2 dentoes the set of all square summable sequences of complex numbers then show that $\langle \{x_n\}, \{y_n\} \rangle = \Sigma x_n \overline{y}_n$ is an inner product on l^2. **6**

b) Verify that C([a, b]) with supremum norm is a normed linear space. **6**

c) In C([0, 1]) with supremum norm compute d(f, g) for f(x) = 1 and g(x) = x. **4**

2. a) Show that a compact subsets of a metric space is closed. **6**

b) State and prove Cauchy-Schwarz inequality. **6**

c) Show that $\mathbb{R}$ with a discrete metric is not separable. **4**

3. a) State and prove Ascoli – Arzela's Theorem. **8**

b) Give an open cover of (–10, 10] that has no finite subcover, consider $\mathbb{R}$ with Euclidean metric. **5**

c) Prove that a totally bounded set is bounded. **3**

4. a) Let m be the Lebesgue measure defined on $\mathbb{R}^n$. Let ε be the collection of all finite unions of disjoint intervals in $\mathbb{R}^n$. Prove that M is a measure on ε. **6**

b) Prove that the outer measure m^* on any subset of $\mathbb{R}^n$ is countably subadditive. **5**

c) Let M_F denote the collection of subsets A of $\mathbb{R}^n$ s.t. distance $D(A_\kappa, A) \to 0$ as $\kappa \to \infty$ where each A_κ is the finite union of disjoint interval. Prove that M_F is a ring. **5**

P.T.O.

5. a) State and prove Holder's inequality. **6**

b) Prove that $\int_E cf\,dm = c\int_E f\,dm$ where f is integrable with respect to Lebesgue measure m over $\mathbb{R}^n$, $c \in \mathbb{R}$ and E is Lebesgue measurable subsets of $\mathbb{R}^n$. **5**

c) If f and g are measurable functions then prove that f + g is measurable. **5**

6. a) State and prove Lebesgue Monotone convergence theorem. **8**

b) Prove that the step functions are dense in $L^P(\mu)$. **6**

c) State Fatou's Lemma. **2**

7. a) State and prove Parseval's Theorem. **8**

b) Show that $\left\{\frac{1}{\sqrt{2\pi}}, \frac{\cos nx}{\sqrt{\pi}}, \frac{\sin mx}{\sqrt{\pi}}\right\}$ where $n, m \in \mathbb{N}$ is an orthonormal sequence in $L^2([-\pi, \pi], m)$. **6**

c) Define mean convergence in $L^2([-\pi, \pi], m)$. **2**

8. a) If $\{U_n\}_{n=1}^{\infty}$ is a sequence of open dense subsets of a complete metric space M then prove that $\bigcap_{n=1}^{\infty} U_n$ is dense in M. Hence prove Baire Category Theorem. **8**

b) Prove that there is no function defined on (0, 1) that is continuous at each rational point of (0, 1) and discontinuous at each irrational point of (0, 1). **6**

c) State Weirstrass Approximation theorem. **2**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT-602 : Differential Geometry
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Let S be a connected n-surface in R^{n+1}. Show that on S, there exists exactly two smooth unit normal vector fields N_1 and N_2. **6**

   b) Show that the speed of geodesic is constant. **4**

   c) Let a, b, c, d $\in R$ be such that ac – b^2 > 0. Show that the maximum and minimum values of the function $g(x_1, x_2) = x_1^2 + x_2^2$ on the ellipse $ax_1^2 + 2bx_1x_2 + cx_2^2 = 1$ are of the form $\frac{1}{\lambda_1}$ and $\frac{1}{\lambda_2}$ where λ_1 and λ_2 are eigen values of the matrix $\begin{pmatrix} a & b \\ b & c \end{pmatrix}$. **6**

2. a) Let U be an open subset of R^{n+1} and $f : U \to R$ be a smooth function. Let $p \in U$ be a regular point of f, and let c = f(p). Show that the set of all vectors tangent to $f^{-1}(c)$ at p is equal to $[\nabla f(p)]^{\perp}$. **6**

   b) Show that the Weingarten map of the n-sphere of radius r oriented by inward normal is multiplication by $\frac{1}{r}$. **5**

   c) Show that the graph of any smooth function $f : R^n \to R$ is an n-surface in R^{n+1}. **5**

P.T.O.

3. a) Let U be an open subset of R^{n+1} and $f : U \to R$ be a smooth function. Let $S = f^{-1}(c)$, $c \in R$ and $\nabla f(q) \neq 0, \forall q \in S$. If $g : U \to R$ is smooth function and $p \in S$ is an extreme point of g on S, then show that there exists a real number λ such that $\nabla g(p) = \lambda \nabla f(p)$. **6**

   b) Show that the tangent space to $SL_2(R)$ at $p = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ can be identified with the set of all 2 × 2 matrices of trace zero. **6**

   c) Sketch the following vector fields on R^2 : X(p) = (p, X(p)) where

   i) X(p)=p

   ii) $X(x_1, x_2) = (x_2, x_1)$. **4**

4. a) Show that the covariant differentiation has the following property :
$(X.Y)' = X'.Y + X.Y'$. **5**

   b) Consider a vector field $X(x_1, x_2) = (x_1, x_2, 1, 0)$ on R^2. For $t \in R$ and $p \in R^2$, let $\phi_t(p) = \alpha_p(t)$ where α is the maximal integral curve of X through p. Show that $F(t) = \phi_t$ is a homomorphism of additive group of real numbers into the invertible linear maps of the plane. **6**

   c) Find the integral curve of the vector field X given by $X(x_1, x_2) = (x_1, x_2, x_2, -x_1)$ through the point (1, 1). **5**

5. a) Let S be an n-surface in R^{n+1}, let $\alpha : I \to S$ be a parametrized curve in S, let $t_0 \in I$ and $v \in S_{\alpha(t_0)}$. Prove that there exists a unique vector field V tangent to S along α, which is parallel and has $V(t_0) = v$. **6**

   b) Let $\alpha(t) = (x(t), y(t))$ be a local parametrization of the oriented plane curve C. Show that $k \circ \alpha = \dfrac{x'y'' - y'x''}{(x'^2 + y'^2)^{\frac{3}{2}}}$. **5**

   c) Show that the 1-form η on $R^2 - \{0\}$ defined by $\eta = \dfrac{-x_2}{x_1^2 + x_2^2} dx_1 + \dfrac{x_1}{x_1^2 + x_2^2} dx_2$ is not exact. **5**

6\. a) Show that the Weingarten map L_p is self-adjoint.
(that is $L_p(v).w = v.L_p(w)$, for all $v, w \in S_p$). **6**

b) Let S denote the cylinder $x_1^2 + x_2^2 = r^2$ of radius r in R^3. Show that α is a geodesic of S if and only if α is of the form $\alpha(t) = (r\cos(at+b), r\sin(at+b), ct+d)$ for some real numbers a, b, c, d. **6**

c) Define the Gauss map and the spherical image of the oriented n-surface S. **4**

7\. a) Prove that on each compact oriented n-surface S in R^{n+1} there exists a point p such that the second fundamental form at p is definite. **6**

b) Let C be a connected oriented plane curve and let $\beta : I \to C$ be a unit speed global parametrization of C. Show that β is either one to one or periodic. **5**

c) Find the curvature of the circle with centre (a, b) and radius r oriented by the outward normal. **5**

8\. a) State and prove Inverse Function Theorem for n-surfaces. **8**

b) Let S be the ellipsoid $\frac{x_1^2}{a^2} + \frac{x_2^2}{b^2} + \frac{x_3^2}{c^2} = 1$, a, b, c all non-zero, oriented by the outward normal. Show that the Gaussian curvature of S is

$$K(p) = \frac{1}{a^2b^2c^2\left(\frac{x_1^2}{a^4} + \frac{x_2^2}{b^4} + \frac{x_3^2}{c^4}\right)^2}.$$ **8**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT-604 : Complex Analysis (New Course)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) If z and z' are points in the extended complex plane $\mathbb{C}_\infty$ and $d(z,z')$ denote the distance between the corresponding point z and z' in $\mathbb{R}^3$

$$d(z,z') = \frac{2\,|z - z'|}{[(1+|z|^2)(1+|z'|^2]^{1/2}}.$$ **8**

b) Find the radius of convergence of a power series $\sum_{n=1}^{\infty} \frac{(-1)^n}{n} z^{n(n+1)}$. **4**

c) Define an analytic function. If f : G → $\mathbb{C}$ is differentiable at a point 'a' in G then prove that f is continuous at a. **4**

2. a) If G is open and connected and f : G → $\mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G then prove that f is constnat. **6**

b) Let f(z) = $\sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence R > 0. Then prove that for each k $\geq$ 1 the series $\sum_{n=1}^{\infty} n\, a_n (z-a)^{n-1}$ has radius of convergence R. **5**

c) i) Show that $(\cos z)' = -\sin z$. **2**

ii) Let f : G → $\mathbb{C}$ be analytic and G is connected. Prove that if f(z) is real for all z in G then f is constant. **3**

P.T.O.

3. a) Define Möbius transformation. Prove that every Möbius transformation maps circles of $\mathbb{C}_\infty$ onto circle of $\mathbb{C}_\infty$. **6**

   b) If $f : G \to \mathbb{C}$ is analytic then prove that f preserves angles each pt Z_o of G where $f'(z_0) \neq 0$. **5**

   c) Let $r(t) = e^{it}$ $0 \le t \le 2\pi$. Find $\int_r z^n dz$. **5**

4. a) Let $\phi : [a, b] \times [c, d] \to \mathbb{C}$ be a continuous function and define $g : [c, d] \to \mathbb{C}$ by $g(t) = \int_a^b \phi(s,t)\, ds$. Then prove that g is continuous. Moreover, if $\frac{\partial \phi}{\partial t}$ exists and is a continuous function on $[a, b] \times [c, d]$ then prove that g is continuously differentiable and $g'(t) = \int_a^b \frac{\partial \phi}{\partial t}(s,t)\, ds$. **8**

   b) State and prove Cauchy's estimate. **4**

   c) Evaluate the integral $\int_r \frac{e^{iz} dz}{z^2}$ where $y(t) = e^{it}$ $0 \le t \le 2\pi$. **4**

5. a) State and prove Fundamental Theorem of Algebra. **6**

   b) Let G be a connected open set and let $f : G \to \mathbb{C}$ be an analytic function. If the set $\{z \in G : f(z) = 0\}$ has a limit point in G then prove that there exist a point a in G such that $f^{(n)}(a) = 0$ for each $n \ge 0$. **5**

   c) If $r : [0, 1] \to \mathbb{C}$ is a closed rectifiable curve and $a \notin \{r\}$ then $\frac{1}{2\pi i} \int_r \frac{dz}{z - a}$ is an integer. **5**

6. a) Let G be an open subset of the plane and $f : G \to \mathbb{C}$ an analytic function. If r is a closed rectifiable curve in G such that $\eta(r; w) = 0$ for all w in $\mathbb{C} - G$ then prove that a in $G - \{r\}$

$$\eta(r; a)\, f(a) = \frac{1}{2\pi i} \int_r \frac{f(z)}{z - a} dz.$$ **8**

b) Let G be a region and let f be an analytic function on G and $a_1, \ldots, a_m$ are points in G that satisfy the equation $f(z) = \alpha$. If r is a closed rectifiable curve in G which does not pass through any pt a_k and if $r \approx 0$ then prove that

$$\frac{1}{2\pi i}\int_r \frac{f'(z)}{f(z)-\alpha}dz = \sum_{k=1}^{m} \eta(r\,;a_k)$$

Hence calculate $\int_r \frac{2z+1}{z^2+z+1}dz$ where r is the circle | z | = 2. **8**

7. a) If f has an isolated singularity at a then prove that the point z = a is a removable singularity iff $\lim_{z\to a}(z-a)\,f(z) = 0$. **8**

b) Prove that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4}dx = \frac{\pi}{\sqrt{2}}$. **6**

c) State Residue Theorem. **2**

8. a) Let G be a region in $\mathbb{C}$ and f an analytic function on G. Suppose there is a constant M such that $\lim_{z\to a}\sup |f(z)| \le M$ for all a in $\partial_\infty G$. Prove that $|f(z)| \le M$ for all Z in G. **6**

b) Let $|a|<1$ and $D = \{z : |z|<1\}$ define $\phi_a : D \to D$ s.t $\phi_a(z) = \frac{z-a}{1-\bar{a}z}$ .Prove that ϕ_a is a one-one map of D onto itself, the inverse of ϕ_a is ϕ_{-a}, ϕ_a maps ∂D onto ∂D. **5**

c) Let p(z) be a polynomial of degree n and let R > 0 be sufficiently large so that P never vanishes in $\{z : |z| \ge R\}$. If r(t) = Re^{it} $0 \le t \le 2\pi$ then prove that $\int_r \frac{P'(z)}{P(z)}dz = 2\pi i n$. **5**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT-706 : Numerical Analysis (Old)

Time : 3 Hours Max. Marks : 80

***N.B.** : 1) Answer **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*
*3) Use of unprogrammable, scientific calculator is **allowed**.*

1. A) Assume that $g \in C[a,b]$. Prove that if the range of the mapping y=g(x) satisfies $a \le y \le b$ for all a < x < b then g has a fixed point in [a, b]. **6**

   B) Investigate the nature of iteration when $g(x) = 2(x-1)^{1/2}$ for $x \ge 1$. Use P_0 = 2.5 and compute P_1, P_2, P_3, P_4. **5**

   C) Find the root of X sin(x) – 1 = 0 in the interval [0, 2] by using false position method. **5**

2. A) Assume that $f \in C^2[a,b]$ and there exists number $P \in [a,b]$ where f(P) = 0. If $f'(P) \ne 0$ then prove that there exists a $\delta > 0$ such that the sequence $\{P_k\}_{k=0}^{\infty}$ defined by iteration $P_k = P_{k-1} - \frac{f(P_{k-1})}{f'(P_{k-1})}$ for k = 1, 2, ...converges to P for any initial approximation $P_0 \in [P-\delta, P+\delta]$. **6**

   B) Let f(x) = x^3 – 3x – 2
      i) Find Newton-Raphson formula.
      ii) Start with P_0 = 2.1 and compute P_1, P_2, P_3.
      iii) Is the sequence converging quadratically or linearly ? **5**

   C) Solve the system of equations

$$\begin{bmatrix} 2 & 1 & 1 & -2 \\ 4 & 0 & 2 & 1 \\ 3 & 2 & 2 & 0 \\ 1 & 3 & 2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -10 \\ 8 \\ 7 \\ -5 \end{bmatrix}$$

   using the Gauss elimination method with partial pivoting. **5**

P.T.O.

3. A) Explain Gaussian Elimination method for solving a system of m equations and n unknowns. **6**

B) Compute the divided difference table for $f(x) = x^3 - 4x$.

x :	1	2	3	4	5	6
f(x):	–3	0	15	48	105	192

Write down Newton's polynomial $P_3(x)$. **5**

C) Find a real root of the equation $x^3 - 2x - 5 = 0$ by using bisection method. (Correct upto 3 decimal places). **5**

4. A) Assume that $f \in C^{N+1}[a,b]$ and $x_0, x_1, \ldots x_N \in [a, b]$ are N + 1 nodes. If $x \in [a,b]$ then prove that $f(x) = P_N(x) + E_N(x)$.

Where $P_N(x)$ is a polynomial that can be used to approximate f(x) and $E_N(x)$ is the corresponding error in the approximation. **8**

B) Consider the system :

$2x + 8y - z = 11$

$5x + y + z = 10$

$-x + y + 4z = 3$

use Gauss-Seidal iteration start with $P_0=0$, to find P_k (k = 1, 2, 3). Will this iteration convergence to the solution ? **8**

5. A) Assume that $f \in C^5[a,b]$ and that $x-2h, x-h, x, x+h, x+2h \in [a, b]$ prove that

$$f'(x) \approx \frac{-f(x+2h)+8(x+h)-8f(x-h)+f(x-2h)}{12h}.$$ **8**

B) Find the triangular factorization A = LU for the matrix $A = \begin{bmatrix} 1 & 2 & 0 & -1 \\ 2 & 3 & -1 & 0 \\ 0 & 4 & 2 & -5 \\ 5 & 5 & 2 & -4 \end{bmatrix}$. **8**

6. A) Assume that $x_j = x_0 + h_j$ are equally spaced nodes and $f_j = f(x_j)$. Derive the quadrature formula $\int_{x_0}^{x_3} f(x)dx \approx \frac{3h}{8}(f_0 + 3f_1 + 3f_2 + f_3)$. **8**

B) Find the Jacobian matrix J(x, y, z) of order 3 × 3, at the points (1, 3, 3) and (1, 2, 3) for the three functions

$f_1(x, y, z) = x^3 - y^2 + y - z^3 + z$

$f_2(x, y, z) = xy + y^2z + xz$

$f_3(x, y, z) = \frac{y}{xz}$. **8**

7. A) Use Euler's method to solve initial value problem $y' = x^2 + y$ over [0, 0.1] with y (0) = 1. Compute y_1, y_2 with h = 0.05. **8**

B) Use the Runge-Kutta fourth order method to find the value of y when x = 1. Given that $y(0) = 1, \frac{dy}{dx} = \frac{y - x}{y + x}$. **8**

8. A) Use power method to find the dominant eigen value and eigen vector for the matrix $A = \begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$. **8**

B) Use Householder's method to reduce the following symmetric matrix to trigonal form

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}.$$ **8**

Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT – 801 : Field Theory
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

*N.B. : 1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) Let $f(x) = a_0 + a_1x + ...+a_{n-1}x^{n-1}+x^n \in \mathbb{Z}[x]$ be a monic polynomial. If f(x) has a root $a \in \mathbb{Q}$, then prove that $a \in \mathbb{Z}$ and a divides a_0. **5**

b) If rither a = b or a + b = –z, then show that the polynomial $x^3 + ax^2 + bx + 1 \in \mathbb{Z}[x]$ is reducible over $\mathbb{Z}$. **5**

c) Find the splitting field of $x^2 + 2x + 2 \in \mathbb{Z}_3[x]$ over $\mathbb{Z}_3$, also exhibit all the elements of this field. **6**

2. a) If E is finite extension of the field F and K is finite extension of E, then prove that K is also finite extension of F. **6**

b) Find the minimal polynomial of $\sqrt{2}+\sqrt[4]{3}$ over $\mathbb{Q}(\sqrt{3})$. **5**

c) Find the smallest extension of $\mathbb{Q}$ having all roots of $x^4 + 1$. **5**

3. a) Let E be an algebraic extension of a field F, and let $\sigma : F \to L$ be an embedding of F into an algebraically closed field L, then prove that σ can be extended to an embedding $\eta : E \to L$. **6**

b) Find the smallest normal extension of $\mathbb{Q}\left(2^{1/4}, 3^{1/4}\right)$ in $\overline{\mathbb{Q}}$ (upto isomorphism). **4**

c) For each prime p and each positive integer n $\geq$ 1, prove that the roots of $x^{p^n} - x \in \mathbb{Z}[x]$ in its splitting field over $\mathbb{Z}_p$ are all distinct and form a field F with p^n elements. **6**

P.T.O.

4. a) Prove that a group of automorphism of a field F with p^n elements is cyclic of order n and is generated by ϕ, where $\phi : F \to F$ is given by $\phi(x) = x^p, x \in F$. **8**

b) Let $\mathbb{Q}(\sqrt[3]{2}, w)$ be an extension of $\mathbb{Q}$, where w^3=1 with $w \neq 1$. Is $\mathbb{Q}(\sqrt[3]{2}, w)$ a finite separable extension of $\mathbb{Q}$? If yes find $\alpha \in \mathbb{Q}(\sqrt[3]{2}, w)$ such that $\mathbb{Q}(\alpha) = \mathbb{Q}(\sqrt[3]{2}, w)$. Justify your answers. **8**

5. a) Let F be a finite field of characteristic p, show that each element a of F has a unique p^{th} root $\sqrt[p]{a}$ in F. **5**

b) Find the basis of $\mathbb{Q}(\sqrt[4]{2}, i)$ over $\mathbb{Q}$. **5**

c) Show $\mathbb{R}(\sqrt{-5})$ is normal extension of $\mathbb{R}$. Is $\mathbb{R}(\sqrt{-5})$ a Galois extension of $\mathbb{R}$? If yes find Galois group $G\left(\mathbb{R}(\sqrt{-5})/\mathbb{R}\right)$. **6**

6. a) Let E be a finite separable extension of a field F. If F is a fixed field of G(E/F), then prove that E is a normal extension of F. **6**

b) Find the group of $\mathbb{Q}$-automorphisms of $\mathbb{Q}(\sqrt[3]{2})$. **5**

c) Find the Galois group of f(x) = $(x^2 - 2)(x^2 - 3)$ over $\mathbb{Q}$. **5**

7. a) If $f(x) \in F[x]$ has r-distinct roots in its splitting field E over F. Then prove that the Galois group G(E/F) of f(x) is a subgroup of the group of symmetries S_r. **6**

b) Find Galois group $G\left(E/\mathbb{Q}\right)$, where E is splitting field of $x^4 - 2 \in \mathbb{Q}[x]$ over $\mathbb{Q}$. Find all subgroups of $G\left(E/\mathbb{Q}\right)$ of order 4. Find fixed field of the subgroup $H = \{\sigma_0, \sigma, \sigma^2, \sigma^3\}$ where $\sigma(\sqrt[4]{2}) = i\sqrt[4]{2}$ and $\sigma(i) = i$ and σ_0 is identity automorphism. **10**

8. a) Write a note on the problem of solving polynomials by radicals with reference to Galois theory. **5**

b) Find $G(E/K)$, where $E = \mathbb{Q}(\sqrt[3]{2}, w), K = \mathbb{Q}(w)$. **6**

c) Show that $\mathbb{Q}(\sqrt[4]{3})$ is not normal extension of $\mathbb{Q}$. Is it a separable extension of $\mathbb{Q}$? Justify your answer. **5**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT-806 : Lattice Theory
(2005 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. : *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let $\mathbb{N}_0$ be the set of all non-negative integers. Define $m \leq n$ if there exists $k \in \mathbb{N}_0$ such that $n = km$ (that is m divides n). Prove that $\mathbb{N}_0$ is a lattice under this relation. **5**

   b) Let L be a lattice. Prove that an ideal I is a prime ideal of L if and only if there is onto meet-homomorphism $\phi : L \to C_2$ such that $I = \phi^{-1}(0)$. **5**

   c) Prove that Con(L), the set of all congruence relations of a lattice L, is a lattice. **6**

2. a) Prove that a lattice L can be embedded in the ideal lattice Id (L) and tthis embedding is onto if L is finite. **8**

   b) Define homomorphism of lattices and prove that a homomorphic image of a lattice L is isomorphic to a suitable quotient lattice of L . **8**

3. a) Prove that for a pseudocomplemented lattice L, the set $S(L) = \{a^* | a \in L\}$ is a lattice. **4**

   b) Let L be a distributive lattice with 0. Show that Id (L), the ideal lattice of L, is pseudocomplemented. **4**

   c) Prove that a lattice is modular if and only if it satisfies the condition (*) : if $a \wedge x = a \wedge y$ and $a \vee x = a \vee y$ for $x \leq y$ then $x = y$. **8**

P.T.O.

4. a) Let ϕ and χ be two one-to-one and onto isotone maps such that the composite maps $\phi \circ \chi$ and $\chi \circ \phi$ are identity. Then prove that both ϕ and χ are isomorphisms. **5**

   b) Prove that in a distributive lattice L, the element $a \neq 0$ is join-irreducible if and only if $L - [a)$ is a prime ideal. **5**

   c) Prove that in a distributive lattice L, if the ideals $I \vee J$ and $I \wedge J$ are principal then so are I and J. **6**

5. a) Let L be a pseudocomplemented lattice. Show that $a^{\perp} = \{x \in L \mid x \wedge a = 0\}$ is an ideal of L. **4**

   b) Prove that a maximal ideal of a distributive lattice is prime but not conversely. **5**

   c) State and prove Nachbin theorem. **7**

6. a) Prove that every finite distributive lattice is isomorphic to ring of sets. **8**

   b) State and prove Stone's Theorem for distributive lattices. **8**

7. a) State and prove Jordan-Hölder Theorem for semimodular lattices. **7**

   b) Prove that every modular lattice satisfies the upper and the lower covering conditions. **5**

   c) Prove that every prime ideal of a Boolean lattice is maximal. **4**

8. a) State and prove Fixed-Point Theorem for complete lattices. **6**

   b) Define conditionally complete lattice and illustrate with an example. Prove that every conditionally complete lattice is complete, if it has the least and greatest element. **6**

   c) Prove that in a distributive lattice, the ideal generated by a dual atom is prime. **4**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2013
MATHEMATICS
MT–502 : Advanced Calculus
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Attempt* ***any 5*** *questions.*
2) Figures at ***right*** *indicate* ***full*** *marks.*

1. A) Show by example that a function of two variable may be continuous in each variable separately and yet discontinuous as a function of two variables separately. **6**

   B) Compute directional derivative of $f(x) = ||x||^2$ for all x in $\mathbb{R}^n$. **5**

   C) Prove or disprove : Existence of directional derivative implies the continuity of functions. **5**

2. A) State and prove sufficient condition of differentiability of scalar field. **6**

   B) State and prove matrix form of chain rule of vector fields. **10**

3. A) Calculate the line integral of vector field $\bar{f}$ along parabola $y = x^2$ where $f(x,y) = (x^2 - 2xy)\ \bar{i} + (y^2 - 2xy)\ j$ from (–2, 4) to (2, 4). **5**

   B) Show that work done by constant force depends on the endpoints and not on the path joining them. **6**

   C) State necessary and sufficient conditions for a vector field to be gradient. **5**

4. A) State and prove first fundamental theorem for line integrals. **10**

   B) Evaluate $\iint\limits_Q (x \sin y - ye^x)\, dx\, dy$ where $Q = [-1, 1] \times [0, \frac{\pi}{2}]$. **6**

P.T.O.

5. A) Change the order of integration $\int_1^2 \left[\int_{2-x}^{\sqrt{2x-x^2}} f(x, y)\, dy \right] dx$ and sketch the region. **6**

B) State and prove Green's theorem for plane regions bounded by piecewise Smooth Jordan curve. **10**

6. A) Evaluate $\iiint_S x\, y^2 z^3\, dxdydz$ where S is the solid bounded by the surface z = xy and the planes y = x, x = 1 and z = 0. **8**

B) Define fundamental vector product of surface described in parametric form. **6**

C) Define area of parametric surface. **2**

7. A) Show that divergence of curl of vector field is zero. **5**

B) Find curl and divergence of $F(x, y) = \frac{-y}{x^2+y^2}\, i + \frac{x}{x^2+y^2}\, j$ **5**

C) Determine Jacobian matrix for $F(x, y, z) = e^{xy}\, \bar{i} + \cos xy\, \bar{j} + \cos xz^2\, \bar{k}$. **6**

8. A) State and prove divergence theorem. **10**

B) Let $f : \mathbb{R}^B \to \mathbb{R}^2$ and $g : \mathbb{R}^3 \to \mathbb{R}^3$ be vector fields defined by **6**

$f(x, y, z) = (x^2 + y^2)\, i + (2xyz)\, \bar{j}$

$g(x, y, z) = x^2 \sin y\, \bar{i} + y^2 \sin z\, \bar{j} + \bar{k}$

Find Jacobian matrix Dh [1, 4, 6] where h (u, v, w) = f[g(u, v, w)].

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2013
MATHEMATICS
MT-503 : Linear Algebra
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

Instructions : *1) Answer **any five** questions.*
*2) Marks at **right** indicate **full** marks.*

1. A) "The set $A = \{ P(x)/P(x) = P(1-x) \text{ for all } x \}$ is subspace of R[x], where R[x] is vector space over $\mathbb{R}$". Comment on above statement. (true or false – Justify). **5**

   B) Prove that a linearly independant subset of a finite dimensional vector space can be extended to form a basis of vector space. **8**

   C) Show that linear transformation over vector spaces maps zero element to zero element. **3**

2. A) Determine whether the mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by **6**

   T(x, y, z) = (2x + 2y – z, 2x + 5y – 2z, 4x + 4y – 2z) is linear transformation or not. If yes, find Kernel and range.

   B) State and prove second isomorphism theorem for vector spaces. **6**

   C) Show that with usual addition and scalar multiplication over rationals, $Q[\sqrt{3}] = \{ a + b\sqrt{3} \mid a, b \in Q \}$ is a vector space over Q. **4**

3. A) Find the matrix representation of linear transformation T with respect to ordered basis

   $\{[0\ \ 11]^t, [1.0\ \ 1]^t, [1\ \ 10]^t\}$ and

   $\{[1\ \ 1]^t, [1, -1]^t\}$ where T is given by

   $$T\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x + 4y - 2z \\ x - y + z \end{bmatrix}$$ **6**

P.T.O.

B) Find eigen values and eigen vectors for matrix $A = \begin{bmatrix} 4 & 2 & 2 \\ 3 & 3 & 2 \\ -3 & -1 & 0 \end{bmatrix} \in \mathbb{R}^{3\times 3}$. **6**

C) Define companion matrix with example. **4**

4. A) State and prove primary decomposition theorem. **10**

B) Prove that eigen vectors corresponding to distinct eigen values of a linear operator are linearly independant. **6**

5. A) Determine Jordan Cannonical form of A in $\mathbb{R}^{4\times 4}$ given by **6**

$$A = \begin{bmatrix} -2 & 5 & 1 & 0 \\ -2 & 4 & 1 & 0 \\ -1 & 2 & 1 & 0 \\ -1 & 2 & 0 & 1 \end{bmatrix}$$

B) State and prove Riesz representation theorem for inner product spaces. **8**

C) Define complete orthonormal set. **2**

6. A) Using Gram-Schmidt orthonormalization process to obtain an orthonormal basis spanned by following vectors in the standard inner product space. A = {[1, –2, 2, 3], [1, 1, 3, –5], [3, 2, 1, –1]}. **8**

B) State and prove Schur's theorem for triangulable linear opertor on finite dimensional inner product space V. **8**

7. A) Show that bilinear form is reflexive if and only if it is either symmetric or alternating. **8**

B) Define the following terms : **8**

a) hyperbolic pair

b) hyperbolic plane.

8. A) State and prove spectral theorem for triangular linear operator on finite dimensional innerproduct space V over F. **8**

B) Show that Jordan chain consists of linearly independant vectors. **6**

C) Define rational cannonical form. **2**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2013
MATHEMATICS (2008 Pattern)
MT – 504 : Number Theory

Time : 3 Hours   Max. Marks : 80

N.B. *: 1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. a) State and prove Wilson's theorem. **6**

b) If (a, m) =1 then prove that there is an x such that ax $\equiv$1 (mod m). Further show that any two such x are congruent (mod m). **4**

c) Prove that any number that is a square must have one of 0, 1, 4, 5, 6, 9 for its units digit. **6**

2. a) Let m be a positive integer, then prove that

$$\phi(m) = \prod_{p/m}\left(p^{\alpha} - p^{\alpha-1}\right) = m \prod_{p/m}\left(1 - \frac{1}{p}\right),$$ where $m = \prod p^{\alpha}$ is canonical factorization of m. **6**

b) Find the least positive integer x such that x$\equiv$5 (mod 7), x$\equiv$7(mod 11) and x $\equiv$ 3(mod 13). **6**

c) Find all solutions of 64x $\equiv$ 83 (mod 105). **4**

3. a) If x and y are any real numbers, prove that

i) [x] + [y] $\leq$ [x + y] $\leq$ [x] + [y] +1 and

ii) $[x] + [-x] = \begin{cases} 0 & \text{if } x \text{ is an integer} \\ -1 & \text{otherwise} \end{cases}$ **6**

b) Let p denote a prime. Then prove that the largest exponent e such that $p^e | n!$ is given by $e = \sum_{i=1}^{\infty} \left[\frac{n}{p^i}\right]$. **6**

c) Find highest power of 7 that divides 1000!. **4**

4. a) Define a divisor function d. If n is positive integer, then prove that $d(n) = \prod_{p^{\alpha} \| n} (\alpha + 1)$ in usual notations. **6**

b) Evaluate $\sum_{j=1}^{\infty} \mu(j!)$. **5**

c) Prove that all the solutions of 3x + 5y = 1 can be written in the form x = 2+ 5t, y = – 1 – 3t. **5**

5. a) If an irreducible polynomial p(x) divides a product f(x) g(x), then prove that p(x) divides at least one of the polynomials f(x) and g(x). **5**

b) Prove that an algebraic number ξ satisfies a unique irreducible monic polynomial equation g(x) = 0 over $\mathbb{Q}$. Further show that every polynomial equation over $\mathbb{Q}$ satisfied by ξ is divisible by g(x). **6**

c) Find the minimal polynomial of $\alpha = 1 + \sqrt{2} + \sqrt{5}$ over $\mathbb{Q}$. Is this number an algebraic integer ? Justify. **5**

6. a) If α is any algebraic number, then prove that there is a rational integer b such that $b\alpha$ is an algebraic integer. **5**

b) Define norm $N(\alpha)$ of a number in $\mathbb{Q}(\sqrt{m})$. Prove that if α is an integer $\mathbb{Q}(\sqrt{m})$, then $N(\alpha) = \pm 1$ if and only if α is a unit. **6**

c) Is $\mathbb{Q}(\sqrt{m})$ unique factorization domain for every integer m ? Justify your answer. **5**

7. a) If p is an odd prime and (a, 2p) = 1, then prove that $\left(\frac{a}{p}\right) = (-1)^{t}$, where

$t = \sum_{j=1}^{\left(\frac{p-1}{2}\right)} \left[\frac{ja}{p}\right]$, also show that $\left(\frac{2}{p}\right) = (-1)^{(p^2-1)/8}$. **8**

b) Prove that 3 is quadratic residue of 13, but it is quadratic non-residue of 7. **5**

c) Show that 3 is not a prime in $\mathbb{Q}\left(\sqrt{6}\right)$. **3**

8. a) If Q is odd and Q > 0, then prove that $\left(\frac{-1}{Q}\right) = (-1)^{(Q-1)/2}$, where $\left(\frac{-1}{Q}\right)$ denote Jacobi symbol. **6**

b) Find the value of $\left(\frac{2}{11}\right)$, where $\left(\frac{2}{11}\right)$ denote Legendre symbol. **5**

c) Let m $\equiv$ 1 (mod 4), prove that $a + b\left(\frac{1+\sqrt{m}}{2}\right)$ is an integer, where a and b are rational integers. **5**

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Seat No.	

M.A./M.Sc. (Semester – I) Examination, 2013
MATHEMATICS
MT – 505 : Ordinary Differential Equations
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. *: i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) If $Y_1(X)$ and $Y_2(X)$ are any two solutions of equation $Y'' + P(X)Y' + Q(X)Y = 0$ on [a, b] then prove that their Wronskian $W = (Y_1, Y_2)$ is identically equal zero or never zero on [a, b]. **5**

b) Find the general solution of $X^2Y'' + 2XY' - 2Y = 0$. **5**

c) Verify that $Y_1 = X$ is one solution of $Y'' - \frac{x}{x-1}Y' + \frac{x}{x-1}Y = 0$ and find Y_2 and general solution. **6**

2. a) Discuss the method of variation of parameters to find the solution of second order differential equation with constant coefficients. **8**

b) Find the general solution of $Y'' - 2Y' + 5Y = 25X^2 + 12$ by using method of undetermined coefficients. **8**

3. A) State and prove Sturm comparison theorem. **8**

B) Verify that origin is regular, singular point and calculate two independent Frobenius series solution for the equation :

$2XY'' + (X+1)Y' + 3Y = 0$ **8**

P.T.O.

4. A) Let u(X) be any non-trivial solution of $u'' + q(X)u = 0$ where q(X) > 0 for all $X > 0$.

If $\int_1^\infty q(X)dX = \infty$ then prove that u(X) has infinitely many zeros on the positive X-axis. **8**

B) Find the general solution of $(1+X^2)Y'' + 2XY' - 2Y = 0$ in terms of power series in X. Can you express this solution by means of elementary functions ? **8**

5. A) Find the general solution of the system :

$$\frac{dX}{dt} = 3X - 4Y\,;\ \frac{dY}{dt} = X - Y$$ **8**

B) Locate and classify the singular points on the X-axis of

$$(3X+1)XY'' - (X+1)Y' + 2Y = 0$$ **4**

C) Determine the nature of the point $X = \infty$ for the equation

$$X^2Y'' + XY' + (X^2 - P^2)Y = 0\,.$$ **4**

6. A) Find the general solution near X = 0 of the hyper geometric equation :

$$X(X-1)Y'' + \left[c - (a+b+1)X\right]Y' - abY = 0$$

where a, b and c are constants. **8**

B) Find the exact solution of initial value problem $Y^1 = Y^2$, Y(0) = 1; starting with $Y_0(X) = 1$. Apply Picard's method to calculate $Y_1(X)$; $Y_2(X)$; $Y_3(X)$ and compare it with the exact solution. **8**

7. A) Show that the function f(X, Y) = XY^2 satisfies Lipschitz condition on any rectangle $a \leq X \leq b$ and $c \leq Y \leq d$; but it does not satisfy a Lipschitz condition on any strip $a \leq X \leq b$ and $-\infty < Y < \infty$. **10**

B) Solve the following initial value problem

$$\frac{dY}{dX} = Z \quad Y(0) = 1$$
$$\frac{dZ}{dX} = -Y \quad Z(0) = 0$$
6

8. A) Find the general solution of $(1 - X^2)Y'' - 2XY' + P(P+1)Y = 0$ about X = 0 by power series method. **8**

B) If m_1 and m_2 are roots of the auxiliary equation of the system :

$$\frac{dX}{dt} = a_1X + b_1Y \; ; \; \frac{dY}{dX} = a_2X + b_2Y$$

Which are real, distinct and of same sign, then prove that the critical point (0, 0) is a node. **8**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT – 601 : General Topology (New) (2008 Pattern)

Time : 3 Hours Max. Marks :80

Instructions : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Let X = {a, b, c}
Define all topologies on X which contains exactly five elements in each topology. **6**

b) Show that both, lower limit topology and k-topology are finer than usual topology on $\mathbb{R}$. **6**

c) Suppose X and Y are two topological spaces. Show that the collection $S = \{\pi_1^{-1}(U)/U \text{ open in } X\} \cup \{\pi_2^{-1}(V)/V \text{ open in } Y\}$ forms a subbasis for product topology on X × Y. **4**

2. a) Let A be subset of a topological space X and A' denote the set of all limit points of X then with usual notations, prove that $\overline{A} = A \cup A'$. Hence find $\overline{Q}$ in usual topology on $\mathbb{R}$. **6**

b) Show that, a topological space X is Hausdorff iff $\Delta = \{x \times x / x \in X\}$ is closed in X × Y. **6**

c) Find interior and closures of following subsets of $\mathbb{R}^2$:

i) $A = \{(x, y)/x \in R, y = 0\}$

ii) $B = \{(x, y)/0 < x^2 + y^2 \leq 1\}$ **4**

3. a) Define homeomorphism between two topological spaces X and Y. Also give an example of a non-homeomorphic function between subset of real numbers and unit ball in $\mathbb{R}^2$. **5**

b) Let A be any set, X and Y be two topological spaces with $f : A \to X \times Y$ defined by $f(a) = (f_1(a), f_2(a))$ then show that f is continuous iff f_1 and f_2 are continuous, where $f_1 : A \to X$ and $f_2 : A \to Y$. **6**

P.T.O.

c) Let A be any set and X be a topological space. Define a quotient map between A and X and show that, $T = \{U \subset A \,/ p^{-1}(U) \text{ open in } X\}$ forms a quotient topology on A. **5**

4. a) Let (X, T) be a topological space and $A \subset B \subset \overline{A} \subset X$. Show that, if A is connected then $\overline{A}$ is connected. **5**

b) Show that arbitrary product of connected spaces is connected. **6**

c) Give an example of connected but not path connected set. **5**

5. a) Show that a compact topological space is limit point compact. Is converse true ? Justify. **6**

b) Let $f : X \to Y$ be a continuous map between two topological spaces. Show that f (X) is connected if X is connected space. **5**

c) State and prove Tube lemma. **5**

6. a) Show that every second countable space is first countable also. Whether canvers holds ? Justify. **5**

b) Define Lindelöf space

i) Is any subspace of a Lindelöf space is Lindelöf ?

ii) Is prove continuous image of Lindelöf space is Lindelöf ?

iii) What can you say about finite product of Lindelöf spaces ? Give counter example if not true. **6**

c) Prove that every second countable space is separable. **5**

7. a) Show that Hausdorffness is a hereditory property and it is finitely multiplicative. **6**

b) Show that metric space is normal. **4**

c) Give examples of following topological spaces :

i) Completely regular but not normal

ii) Regular but not completely regular

iii) Hausdorff but not regular. **6**

8. a) State and prove Urysohn's lemma. **12**

b) State Tietze extension theorem. **2**

c) State Lebesque covering lemma. **2**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT-601 : Real Analysis – II
(Old)

Time : 3 Hours Max. Marks : 80

N.B. *: 1) Attempt **any five** questions.*
*2) **All** questions carry **equal** marks.*
*3) Figures to the right indicate **full** marks.*

1. a) Define a function of bounded variation and with usual notations prove that, for $a \leq c \leq b$. **6**

$V_a^c f + V_c^b f = V_a^b f$

b) Show that, BV [a, b] is complete space under the norm $\|f\|_{BV} = |f(a)| + V_a^b f$. **6**

c) Give statements of

i) Jordan's theorem

ii) Helly's first theorem. **4**

2. a) For a bounded and real valued function f and α, define Riemann Steiltje's integral of f over α. **8**

Whether such integral exists for all bounded f ? Justify.

b) Show that addition and multiplication of two Riemann-Steiltje's integrable functions is also Riemann-Steiltje's integrable. **6**

c) State Riesz representation theorem. **2**

3. a) Let $f = [0, 2\pi] \to \mathbb{R}$ with $f(x) = \pi - x$ and $f(0) = f(2\pi) = 0$. Show that Fourier series for extended f is $2\sum_{n=1}^{\infty} \frac{\sin n x}{n}$. **6**

b) State and prove Fejer's theorem. **6**

c) Find Dirichlet and Fejer's Kernel for $f = \mathbb{R} \to \mathbb{C}$ defined by

$f(t) = \sum_{k=-\infty}^{\infty} C_k e^{ikt}$ where $C_k = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(t) e^{-ikt} dt$. **4**

4. a) Define Lebesgue outer measure (m^*) of E subset of $\mathbb{R}$ and show that m^* is non-negative, monotonic and translation invariant. **8**

b) With usual notations prove that M forms an algebra. **6**

c) State Vitali's covering lemma. **2**

5. a) Give an example of a non-measurable set. **8**

b) Show that a Riemann integrable function is a Lebesgue integrable also. But converse does not hold. **8**

6. a) State and prove Monotone convergence theorem. **6**

b) For a non-negative and measurable function f,

show that $\int f = 0$ iff $f \equiv 0$. (a. e). **6**

c) Whether strict inequality hold is Fatou's lemma ? Justify. **4**

7. a) State and prove Lebesgue dominated convergence theorem. **8**

b) Suppose $1 < p, q < \infty$ with $\frac{1}{p} + \frac{1}{q} = 1$

If $f \in L_p(E)$, $g \in L_q(E)$, then show that $fg \in L_1(E)$

Moreover $\left| \int_E fg \right| \leq \int_E |fg| \leq \|f\|_p \, \|g\|_q$. **8**

8. a) What are derived numbers ? Find derived numbers for $f(x) = x \sin\left(1/x\right)$ on $[-1, 1]$. **8**

b) Define absolute continuity and show that if,

1) $f \in AC\,[a, b]$ then $f \in C\,[a, b] \cap BV\,[a, b]$

2) $f \in AC\,[a, b]$ iff $V(x) = V_a^x f \in AC\,[a, b]$. **8**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT-603 : Groups and Rings
(New) (2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B.: *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let H be a non-empty finite subset of a group G.
Prove that if H is closed under the operation of G then H, is a subgroup of G.
What happens if H is not finite ? **5**

b) i) Define the dihedral group D_n. Find the centre of D_3 and D_4. **3**

ii) Define the centralizer of an element 'a' in a group G. Find the centralizer of a = (1 2 3 4) in D_4. **3**

c) D_4 has seven cyclic subgroups. List them. Find a subgroup of D_4 of order 4, which is not cyclic ? **5**

2. a) Let G = $\langle a \rangle$ be a cyclic group of order n. Prove that G = $\langle a^k \rangle$ iff g.c.d (k, n) = 1. **5**

b) i) Prove that no group is the union of two proper subgroups. Does the result remains true if "two" is replaced by "three". **3**

ii) Prove that if $a \in G$ is the only element of order 2 in G then a lies in the centre of a group G. **3**

c) Let G be group with exactly two non-trivial proper subgroups. Prove that either G is cyclic and $|G| = pq$, p and q are primes or G is cyclic and $|G| = p^3$ for some prime p. **5**

3. a) If the pair of cycles $\alpha = (a_1, a_2 \ldots a_m)$ and $\beta = (b_1, b_2 \ldots b_n)$ have no entries in common then prove that $\alpha\beta = \beta\alpha$. **5**

P.T.O.

b) Find the total number of odd permutations of order 4 in S_6. List any five. **5**

c) i) Find the group elements α and β so that $|\alpha| = 3, |\beta| = 3$ and $|\alpha\beta| = 5$. **3**

ii) How many elements of order 5 are there in A_6 ? **3**

4. a) Prove that every group is isomorphic to a group of permutation. **5**

b) Determine whether the following pairs of groups are isomorphic ? **5**

i) S_4, D_{12}

ii) Aut Z_6, Aut Z_3.

c) Prove that any subgroup of a group G of index 2 is normal subgroup. Use this to prove that A_4 has no subgroup of order 6. **6**

5. a) Prove that the order of an element of a direct product of a finite number of finite groups is the L.C.M. of orders of the components of the elements. **5**

b) Prove or disprove **3**

i) In a factor group $\frac{G}{H}$ **3**

if $aH = bH$

$\Rightarrow |a| = |b|$.

ii) D_4 is the internal direct product of its two proper subgroups. **3**

c) If $H = \int \beta \in S_5 / \beta(1) = 1, \beta(3) = 3$ then prove or disprove that H is a subgroup of S_5. What is the cardinality of H if $H \leq G$? **5**

6. a) If K is a subgroup of G and N is a normal subgroup of G then prove that

$$\frac{K}{K \cap N} \simeq \frac{KN}{N}.$$ **6**

b) Determine all homomorphic images of D_4. **5**

c) Let $\mathbb{R}^*$ be the group of non-zero real numbers under multiplication and $\mathbb{R}^+$ be the group of positive real numbers.

Show that $\mathbb{R}^+$ is the proper subgroup of $\mathbb{R}^*$ of finite index. **5**

7. a) If m divides the order of a finite abelian group G then prove that G has a subgroup of order m. **6**

b) Let $H \triangleleft S_4$ of order 4 (ie $|H| = 4$) prove that $\frac{S_4}{H} \simeq S_3$. **5**

c) If $G = \left\{ \begin{pmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{pmatrix} \middle| \, a, b, c \in Q - \text{rationals} \right\}$ is a group under matrix multiplication then find $\frac{G}{Z(G)}$. **5**

8. a) State and prove Sylow's first theorem. **6**

b) Find the class equation of D_4. **5**

c) Show that the centre of a group of order 60 cannot have order 4. **5**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT-603 : Group Theory
(Old Course)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Prove that a finite semigroup G is a group if and only if it admits left and right cancellations. **5**

   b) Construct an abelian group G of order K^n in which $x^k = 1$ for all x in G, K and n being given natural numbers. **5**

   c) Prove that no two of the additive groups Z, Q, $\mathbb{R}$ are isomorphic to each other. **6**

2. a) Prove that for $n \geq 3$, the 3-cycles (1 2 3), (1 2 4), --, (1 2 n) generate the alternating group A_n. **5**

   b) If $S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 10 & 1 & 5 & 3 & 4 & 8 & 6 & 2 & 7 & 9 \end{pmatrix}$ **5**

   Find 1) the number of inversions in s

   2) sgn (s)

   Hence determine whether s is even or odd permutation.

   c) With usual notation determine all homomorphisms of D_n into C^x. **6**

3. a) Prove that if H and K are subgroups of a group G then their product HK is a subgroup of G if and only if HK = KH. **6**

   b) Prove that the commutator subgroup of the alternating group A_4 is the Klein's four group V_4. **5**

   c) Prove or disprove

   If a, b $\in$ G with $0(a) < \infty$, $0(b) < \infty$ then $0(ab) < \infty$. **5**

4. a) If H is a subgroup of the group G and L is a subgroup of H then prove that L is a subgroup of G and [G : L] = [G : H] [H : L]. **6**

b) Prove that the converse of Lagrange's theorem hold in S_4. **5**

c) Find the cosets in $\mathbb{R}^3$ of the xy – plane $\{(x, y, 0) | x, y \in \mathbb{R}\}$. Find a system of representaties in above case. **5**

5. a) Prove that the centre of a p-group is always non trivial. **5**

b) Find all conjugate classes of a quaternian group Q_8 and hence find the class equation of Q_8. **6**

c) If all subgroups of the group G are normal in G then show that the commutator [a, b] commutes with each of a and b. **5**

6. a) State and prove the first isomorphism theorem. **6**

b) Prove that for a natural number n, nZ is a maximal subgroup of Z if and only if n is prime. Give an example of an abelian group having no maximal subgroup. **6**

c) Prove the $\mathrm{Inn}(S_n) \simeq S_n \ \ n \geq 3$. **4**

7. a) If the prime power p^{β} divides the order n of a finite group G then prove that G has a subgroup of order p^{β}. **6**

b) Describe upto isomorphism, all finite abelian groups of order 12, 16, 108. **5**

c) Prove or disprove

Any abelian group of order 21 is cyclic. **5**

8. a) If H is a normal subgroup of G and if H and $\frac{G}{H}$ are both solvable then prove that G is solvable. **6**

b) Give an example of a group G and normal subgroups $N_1, ..., N_K$ such that $G = N_1 ... N_K$ and $N_i \cap N_j = \{e\}$ for all i and j $e \neq j$ but G is not the internal direct product of $N_1, ..., N_K$. **5**

c) Is $(Q,+)$ decomposible ? Justify. **5**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT – 605 : Partial Differential Equations
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

Instructions *: i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate maximum marks.*

1. a) Explain the classification of first order partial differential equations as linear, semilinear, and quasi-linear PDE. **4**

b) Explain Charpit's method for finding complete integral of a first order PDE f (x, y, z, p, q) = 0. Further find a complete integral of the PDE $(p^2 + q^2) y - qz = 0$. **8**

c) Find the integral of the Pfaffian differential equation :
yzdx + 2xzdy – 3xydz = 0. **4**

2. a) Show that the general solution of the quasi-linear equation

P (x, y, z) p + Q (x, y, z) q = R (x, y, z), where P, Q, R are continuously differentiable functions of x, y, z is F (u, v) = 0, where F is an arbitrary differentiable function of u, v and u (x, y, z) = c_1, and v (x, y, z) = c_2 are two independent solutions of the system

$$\frac{dx}{P(x, y, z)} = \frac{dy}{Q(x, y, z)} = \frac{dz}{R(x, y, z)}$$ **8**

b) Find the integral surface of the equation $x^3p + y(3x^2 + y)q = z(2x^2 + y)$ which passes through the curve C : $x_0 = 1$, $y_0 = s$, $z_0 = s(1 + s)$. **8**

3. a) Find general solution of the PDE $(xp - yq)z = y^2 - x^2$. **6**

b) Find a complete integral of the following equation by Jacobi's method :
$(p^2x + q^2y) - z = 0$. **4**

P.T.O.

c) Consider the first order quasi-linear PDE $P(x, y, z) Z_x + Q(x, y, z) Z_y = R(x, y, z)$, where P, Q, R have continuous partial derivatives with respect to x, y, z. Let the value $z = z_0$ be prescribed along the initial curve $\Gamma_0 : x = x_0(s), y = y_0(s)$; where x_0, y_0, z_0 are C^1 functions defined along $a \le s \le b$.

If $\frac{dy_0(s)}{ds} P(x_0(s), y_0(s), z_0(s)) - \frac{dx_0(s)}{ds} Q(x_0(s), y_0(s), z_0(s)) \neq 0$, then show that there exists a unique solution z (x, y) defined in some neighborhood of the initial curve Γ_0 which satisfies the PDE and the initial conditions $z(x_0(s), y_0(s)) = z_0(s)$. **6**

4\. a) For a nonlinear first order PDE $f(x, y, z, p, q) = 0$, derive analytic expression for the Monge cone at (x_0, y_0, z_0). Further consider the equation $p^2 + q^2 = 1$. Find the equation of the Monge cone with vertex at (0, 0, 0). **6**

b) Find the characteristic strips of the equation $xp + yq - pq = 0$, and obtain the equation of the integral surface through the curve $C : z = x / 2, y = 0$. **6**

c) Derive the De'Alembert's solution the Cauchy problem :

$u_{tt} - c^2 u_{xx} = 0, -\infty < x < \infty, t > 0$

$u(x, 0) = f(x), u_t(x, 0) = g(x)$. **4**

5\. a) Using the method of separation of variables obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions :

$u(0, t) = u(2, t) = 0, u(x, 0) = \sin^3\left(\frac{\pi x}{2}\right)$, ut (x, 0) = 0. **6**

b) Show that the solution of the following problem, if exists, is unique.

$u_{tt} - c^2 u_{xx} = F(x, t), 0 < x < l, t > 0,$

$u(x, 0) = f(x), 0 \le x \le l$

$u_t(x, 0) = g(x), 0 \le x \le l,$

$u(0, t) = u(l, t) = 0, t \ge 0.$ **6**

c) Find eigen functions and eigen values of the Sturm-Liouville problem :

$$\frac{d^2y}{dx^2} + \lambda y = 0, 0 < x < L,$$

$$y'(0) = y'(L) = 0.$$ **4**

6. a) Find the characteristic lines of the hyperbolic equation $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$. **4**

   b) Reduce the following equation to canonical form :

   $u_{xx} + 2u_{xy} + 4u_{yy} + 2u_x + 3u_y = 0.$ **6**

   c) Reduce the following equation to canonical form and solve.

   $y^2u_{xx} - 2xyu_{xy} + x^2u_{yy} = \frac{y^2}{x}u_x + \frac{x^2}{y}u_y.$ **6**

7. a) Suppose u (x, y) is harmonic in a bounded domain D and continuous in $D \cup B$. Then show that u attains its maximum and minimum on the boundary B of D. **8**

   b) Solve the boundary value problem :

   $u_t = u_{xx}$, $0 < x < l$, $t > 0$

   $u(0, t) = u(l, t) = 0$,

   $u(x, 0) = x(l - x)$, $0 \leq x \leq l$. **8**

8. a) Solve the following diffusion equation using Fourier transform technique :

   $u_t = ku_{xx}$, $-\infty < x < \infty$, $t > 0$;

   $u(x, 0) = f(x)$, $-\infty < x < \infty$. **6**

   b) Find the Fourier transform of $f(x) = e^{\frac{-x^2}{2}}$. **4**

   c) Solve the boundary value problem :

   $u_{xx} + u_{yy} = 0$, $0 < x < a$, $0 < y < b$,

   $u(x, 0) = f(x)$, $0 \leq x \leq a$,

   $u(x, b) = 0$, $0 \leq x \leq a$,

   $u(0, y) = 0$, $0 \leq y \leq b$,

   $u(a, y) = 0$, $0 \leq y \leq b$. **6**

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Seat No.	

M.A./M.Sc. (Semester – II) Examination, 2013
MATHEMATICS
MT-606 : Object Oriented Programming using C++
(2008 Pattern)

Time : 2 Hours Max. Marks : 50

Instructions : *1) Figures at **right** indicate **full** marks.*
*2) Question No. **1** is **compulsory**.*
*3) Attempt **any three** questions out of Question **2, 3, 4** and **5**.*

1. Attempt **any 10** questions. **Each** question carries **2** marks.

i) Define the term encapsulation.

ii) Which of the following are valid variables.

a) 1 result b) 1_result

c) basic payement d) _andy

iii) What is range of following data types

a) char b) short int

c) double d) long int.

iv) Give general syntax of declaring pointer in C++.

v) Write short note on 'delete operator' in C++.

vi) Write a program for 'boolean data type'.

vii) What is reference variables ?

viii) Write output of following sequence of statements :

a) cout. width (3);

cout << 40000;

b) cout. precision (3);

cout. width (6);

cout << 3.34689.

P.T.O.

ix) What is getline function in C++ ?

x) Define static member.

xi) Write syntax for declaring a constructor.

xii) Write down steps of operator over loading.

2. A) What are features of object oriented programming ? **5**

   B) Write a note on applications of object oriented programming. **5**

3. A) Differentiate between a class and structure in C++. **5**

   B) Write a program to demonstrate a scope resolution operator. **5**

4. A) Write a C++ program to find maximum of two numbers. **5**

   B) Write a C++ program to display book information using class object. **5**

5. A) How interactions takes place between the functions in C++ ? **5**

   B) Write a program to find square root of the number using in line function. **5**

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B/I/13/320

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Seat No.

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT-701 : Functional Analysis
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Let M be a closed linear subspace of a normed linear space N. If a norm of a coset x + M in the quotient space N/M is defined by $|||\, x+M \,||| = \inf \{\| x + m \| : m \in M\}$, then prove that N/M is a normed linear space. Further if N Banach, then prove that N/M is also a Banach space. **8**

   b) Give example of a space with two non-equivalent norms on it. Justify. **6**

   c) A linear operator $S : l^2 \to l^2$ is defined by $S(x_1, x_2, ...) = (0, x_1, x_2, ...)$. Find its adjoint S*. **2**

2. a) State and prove Hahn-Banach Theorem. **8**

   b) Show that an operator T on a finite dimensional Hilbert space H is normal if and only if its adjoint T* is a polynomial in T. **6**

   c) Show that the norm of an isometry is 1. **2**

3. a) Let Y be a closed subspace of a normed linear space X. Show that a sequence $\{x_n + Y\}$ converges to x + Y in X/Y if and only if there is a sequence $\{y_n\}$ in Y such that $\{x_n + y_n\}$ converges to x in X. **8**

   b) Show that every positive operator on a finite dimensional Hilbert space has a unique positive square root. **4**

   c) If T is any operator on a Hilbert space H and if α, β are scalars such that $|\alpha| = |\beta|$, then show that $\alpha T + \beta T^*$ is normal. **4**

4. a) If T is an operator on a Hilbert space H, then prove that T is normal if and only if its real and imaginary parts commute. **6**

   b) i) Let X and Y be normed spaces. If X is finite dimensional, then show that every linear transformation from X to Y is continuous. **4**

   ii) Give an example of a discontinuous linear transformation. **4**

   c) Let H be a 2-dimensional Hilbert space. Let the operator T on H be defined by $Te_1 = e_2$ and $Te_2 = -e_1$. Find the spectrum of T. **2**

P.T.O.

5. a) Show that || T* || = || T || and || T*T || = || T ||^2. **6**

b) Let X be a normed space over C. Let $0 \neq a \in X$. Show that there is some functional f on X such that f (a) = ||a|| and ||f|| = 1. **6**

c) Find $M^{\perp}$ if M = {(x, y) : x – y = 0} $\subset R^2$. **4**

6. a) If T is an operator on a Hilbert space H for which $\langle Tx, x \rangle = 0$ for all $x \in H$, then prove that T = 0. **6**

b) Let S and T be normal operators on a Hilbert space H. If S commutes with T*, then prove that S + T and ST are normal. **6**

c) If P and Q are he projections on closed linear subspaces M and N of a Hilbert space H, prove that PQ is a projection if and only if PQ = QP. **4**

7. a) Let H be a Hilbert space and f be a functional on H. Prove that there exists a unique vector y in H such that f (x) = <x, y> for every $x \in H$. **8**

b) Let T be an operator on H. If T is non-singular, then show that $\lambda \in \sigma$ (T) if and only if $\lambda^{-1} \in \sigma (T^{-1})$. **4**

c) Give an example to show that an infinite dimensional subspace of a normed linear space may not be closed. **4**

8. a) State and prove the Closed Graph Theorem. **8**

b) Let T be a normal operator on H with spectrum $\{\lambda_1, \lambda_2, \ldots \lambda_m\}$. Show that T is self-adjoint if and only if each λ_i is real. **4**

c) Show that the unitary operators on a Hilbert space H form a group. **4**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT – 702 : Ring Theory
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Define a unit and zero-divisor in a ring R. If R is the ring consisting of all functions from closed interval [0, 1] to $\mathbb{R}$ (set of all real numbers) under pointwise addition and multiplication of functions, then find units and zero divisors of R. **5**

b) i) Prove that any subring of a field which contains the identity is an integral domain. **3**

ii) If R is the ring of functions from a non-empty set X to a field F, then prove that R contains no non-zero nilpotent element. **3**

c) Find the units in the ring $Z\left[\frac{1+\sqrt{-3}}{2}\right]$. **5**

2. a) If A is a subring and B is an ideal of a ring R, then prove that $\frac{A+B}{B} \simeq \frac{A}{A \cap B}$. **6**

b) Prove or disprove

i) The rings 2Z and 3Z are isomorphic. **5**

ii) The rings Z(x) and Q(x) are isomorphic.

c) Let R = Z(x) be the ring of polynomials in x with integer coefficients and let I be the collection of polynomials whose terms are of degree at least Z together with zero polynomial ?
Prove that I is an ideal of R. Find $\frac{R}{I}$. Is $\frac{R}{I}$ an integral domain ? Justify. **5**

3. a) Define the sum and product of two ideals I and J. Prove that if R is a commutative ring with identity and if I + J = R, then IJ = [n]. Give an example where IJ $\neq$ I$\cap$J. **6**

b) Let R = Z (x) and I = (z, x) be an ideal. Show that I is not the principal ideal of R and find $\frac{R}{I}$. Is I a prime ideal, maximal ideal ? Justify. **6**

c) Let f(x) = $x^4 - 16 \in$ R = Z (x). Use bar notation to denote passage to quotient ring $\frac{Z[x]}{\langle f(x) \rangle}$. Find a polynomial of degree $\leq$ 3 that is congruent to $7\,x^{13} - 11\,x^9 + 5x^5 - 2x^3 + 3$ modula (f(x)). Prove that $\overline{x-2}$ and $\overline{x+2}$ are zero divisors in $\frac{z(x)}{\langle f(x) \rangle}$. **4**

4. a) Let R be an integral domain and let D be a non-zero subset of R that is closed under multiplication. Prove that the ring of fraction $D^{-1}R$ is isomorphic to a subring of the quotient field of R. **6**

b) Let R be a ring with 1 $\neq$ 0. If e is an idempotent element in the ring R with er = re for all r$\in$R, then prove that Re and R (1 – e) are two sided ideals of R and R$\simeq$Re x R (1 – e). What are identities in the subrings Re and R (1 – e) ? **6**

c) Prove that in a Boolean ring a non zero ideal is prime ideal iff it is maximal ideal. **4**

5. a) If a and b are non-zero elements in the commutative ring R such that the ideal generated by a and b is principal ideal (d), then prove that d is the greatest common divisor of a and b. **6**

b) Define the universal side divisor element in a ring. If R is a Euclidean domain but not a field, then prove that there are universal side divisors in R. What happens if R contains no universal side divisors ? Give an example of a ring R in which there are no universal side divisors. **6**

c) Give an example of two elements a and b in a Euclidean domain R with N(a) = N(b) but a and b are not associates (N is a Euclidean Norm function on R). **4**

6. a) Prove that every non zero prime ideal in a principal ideal domain is a maximal ideal. **5**

   b) Show that the additive group (Q, +) has no maximal subgroup but the ring (Q, +, .) has maximal subring. **6**

   c) Let I and J be a prime ideas in a ring R. Prove that $I \cap J$ is a prime ideal iff either $I \subseteq J$ or $J \subseteq I$. **5**

7. a) Prove that in a integral domain a prime element is always an irreducible element. What can you say about converse ? Justify. **6**

   b) Prove that the $Z\left[\sqrt{-5}\right]$ is an integral domain but not a unique factorization domain. Find $\left|\frac{R}{(3)}\right|$; $R = Z\left[\sqrt{-5}\right]$. **6**

   c) Prove or disprove $\frac{Z[i]}{(7)}$ is a field with 49 elements. **4**

8. a) State and prove Eisenstein's criterion for irreducibility of polynomial. **5**

   b) Let R be a unique factorization domain and F be its field of fraction. If P[x] is monic irreducible polynomial in R[x], then prove that it is irreducible in F[x]. Use this to prove that the ring R = Z [2i] is not unique factorization domain. **6**

   c) Prove that the polynomial $x^{n-1} + x^{n-2} + ... + x + 1$ is irreducible over Z if and only if n is a prime integer. **5**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT – 703 : Mechanics
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

***N.B.** : i) Attempt **any five** questions.*
*ii) **All** questions carry **equal** marks.*
*iii) Figures to the **right** indicate **maximum** marks.*

1. a) Explain the principle of virtual work and derive D'Alembert's equations of motion. **8**

b) Write down Lagrenge's equation for the Lagrangian given by $L(q, \dot{q}) = \frac{1}{2} q^2 \dot{q}^2$; and hence solve it. **6**

c) State Hamilton's principle. **2**

2. a) Is $x\dot{x} + y\dot{y} + \dot{x}y + x\dot{y}$ = constant, a holonomic constraint ? Justify your answer. **6**

b) Explain the following terms with examples :

i) holonomic constraint

ii) conservative force. **4**

c) Determine the number of degrees of freedom in case of

i) a particle moving along the plane curve xy = constant

ii) a free particle moving in space

iii) a rigid body. **6**

P.T.O.

3. a) Let $L(q, \dot{q}) = \frac{1}{2}\dot{q}^2 - q\dot{q} + q^2$. Show that the Poisson bracket $\left[p, \dot{q}^2\right] = -2\dot{q}$. **6**

b) Find the values of α and β for which the following equations :

$Q = q^\alpha \cos\beta p, P = q^\alpha \sin\beta p$ represent a canonical transformation. **3**

c) Find the Hamilton's canonical equations for a one-dimensional harmonic oscillator for which the kinetic energy is $\frac{1}{2}m\dot{x}^2$ and the potential energy is $\frac{1}{2}kx^2$. **4**

d) Consider motion of a free particle having mass m in a plane. Express its kinetic energy in terms of plane polar coordinates and their time derivatives. **3**

4. a) If the Hamiltonian H of the system is

$H = q_1p_1 - q_2p_2 - aq_1^2 + bq_2^2$, where a, b are constants, then show that q_1q_2 is a constant of motion. **4**

b) Write a note on Brachistochrone problem. **6**

c) Find the canonical transformation generated by the function

$F_1(q, Q) = \frac{1}{2}mwq^2 \cot Q$, where m, w are constants. **6**

5. a) Suppose f does not depend explicitly on x. Then show that $f - \frac{\partial f}{\partial y'}y' = \text{constant}$. **4**

b) Write Hamilton's equations of motion using Poisson brackets. Show that $\frac{dH}{dt} = \frac{\partial H}{\partial t}$, where H denotes Hamiltonian. **4**

c) Prove that [f, gh] = g[f, h] + h[f, g]. **4**

d) If L is a Lagrangian for a system of n degrees of freedom, satisfying Lagrange's equations of motion, show that $L' = L + \frac{dF}{dt}$, $F = F(q_1, q_2, ..., q_n, t)$, also satisfies Lagrang's equations where F is arbitrary differentiable function of its arguments. **4**

6. a) State and prove Euler's theorem about general displacement of a rigid body. **6**

b) Show that if F and G are two constants of motion then their Poisson brackets [F, G] is also constant of motion. **5**

c) Define infinitesimal rotations. Show that infinitesimal rotations are pseudo-vectors. **5**

7. a) Show that the following transformation is canonical $Q = \log\left(1+\sqrt{q}\,\cos p\right)$, $P = 2(1+\sqrt{q}\cos p)\sqrt{q}\sin p$. **5**

b) Define orthogonal transformations. Does the following transformation represent an orthogonal transformation ? Justify your answer. **5**

$$x' = 2x + 3y + z$$

$$y' = x + 2y - z$$

$$z' = x - z.$$

c) Explain the concept of Euler angles diagrammatically. **6**

8. a) Show that if the law of central force is an inverse square law of attraction, then the path of the particle is a conic. **6**

b) A particle moves along the curve, $\bar{r} = (\cos t)\,\hat{i} + (\sin t)\,\hat{j} + t\hat{k}$, starting at t = 0. Find velocity and acceleration at t = $\pi/2$. **4**

c) Show that a central force motion is always in a plane and conserves angular momentum. **6**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT-704: Measure and Integration
(New) (2008 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. : 1) *Attempt* ***any five*** *questions.*
2) *Figures to the* ***right*** *indicate* ***full*** *marks.*
3) ***All*** *symbols have their* ***usual*** *meanings.*

1. a) If E_i's are with $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu(\bigcap_{i=1}^{\infty} E_i) = \lim_{n\to\infty} \mu E_n$. **6**

b) Define converge in measure. Show that, if f_n converges to f in measure, then there is a subsequence f_{nk} that converges to f almost everywhere. **6**

c) Let $(X, \mathscr{B}, \mu)$ be a measure space and $Y \in \mathscr{B}$. Let $\mathscr{B}_Y$ consist of those sets of $\mathscr{B}$ that are contained in Y and $\mu_Y(E) = \mu(E)$ if $\in \mathscr{B}_Y$. Then show that $(Y, \mathscr{B}_Y, \mu_Y)$ is a measure space. **4**

2. a) If f and g are integrable functions and E is a measurable set then show that **6**

i) $\int_E (c_1 f + c_2 g) = c_1 \int_E f + c_2 \int_E g$.

ii) If $|h| \le |f|$ and h is a measurable then h is integrable.

iii) If $f \ge g$ a.e. then $\int f \ge \int g$.

b) Suppose that to each α in a dense set D of real numbers there is assigned a set $B_\alpha \in \mathscr{B}$ such that $B_\alpha \subset B_\beta$ for $\alpha < \beta$. Then show that there exist a unique measurable extended real valued function f on X such that $f \le \alpha$ on B_α and $f \ge \alpha$ on $X \sim B_\alpha$. **6**

c) Show that if $\{E_i\}$ is a sequence of measurable sets, $m\left(\bigcup_{i=1}^{\infty} E_i\right) < \infty$ and $\lim E_i$ exists, then m ($\lim E_i$) = lim m (E_i). **4**

P.T.O.

3. a) State and prove Fatou's Lemma. **6**

b) If f and g are nonnegative measurable functions and a and b are nonnegative constants, then show that $\int af + bg = a\int f + b\int g$. **6**

c) Let f be a continuous function and g a measurable function. Show that the composite function of $f \circ g$ is measurable. **4**

4. a) Let (x, $\mathscr{B}$) be a measurable space and $< \mu_n >$ a sequence of measures on $\mathscr{B}$ such that for each $E \in \mathscr{B}$, $\mu_{n+1}E \geq \mu_n$ and $\mu E = \lim \mu_n E$. Then show that μ is a measure on $\mathscr{B}$. **6**

b) State and prove Hahn Decomposition Theorem. **6**

c) Find the cardinality of the class of measurable sets. **4**

5. a) Let $(X, \mathscr{B}, \mu)$ be a σ- finite measure space, and let ν be a measure defined on $\mathscr{B}$ which is absolutely continuous with respect to μ. Then prove that there is a nonnegative measurable function f such that for each set E in $\mathscr{B}$ we have $\nu E = \int_E f d\mu$. **6**

b) Let F be a bounded linear functional on $L^p(\mu)$ with $1 \leq p < \infty$ and μ a σ- finite measure. Then show that there is a unique element g in L^q where $1/p + 1/q = 1$, such that $F(f) = \int fg d\mu$ with $||F|| = ||g||_q$. **6**

c) Let [a, b] be a finite interval and let $f \in L(a, b)$ with indefinite integral F, then show that $F' = f$ a.e. in (a, b). **4**

6. a) Let $(X, \mathbb{B}, \mu)$ be a finite measure space and g an integrable function such that for some constant M, $\left|\int g\phi d\mu\right| \leq M\|\phi\|_p$ for all simple functions ϕ then show that $g \in L^q$. **8**

b) Show that the class $\mathscr{B}$ of μ^*-measurable sets is a σ- algebra. Further, if $\bar{\mu}$ is μ^* restricted to $\mathscr{B}$ then prove that $\bar{\mu}$ is complete measure on $\mathscr{B}$. **8**

7\. a) Define a measure on an algebra $\mathcal{G}$. If $A \in \mathcal{G}$ then show that A is measurable with respect to μ^*. **6**

b) Define product measure. Let E be a measurable subset of X × Y such that $\mu \times \nu(E)$ is finite. Then show that for almost all x the set E_x is a measurable subset Y. **6**

c) If E and F are disjoint sets then show that

$\mu_* E + \mu_* F \leq \mu_*(E \cup F) \leq \mu_* E + \mu^* F \leq \mu^*(E \cup F) \leq \mu^* E + \mu^* F$. **4**

8\. a) If μ^* is a Caratheodory outer measure with respect to Γ then prove that every function in Γ is μ^*-measurable. **6**

b) Let μ be a finite measure defined on a σ-algebra $\mathcal{M}$ which contains all the Baire sets of locally compact space X. If μ is inner regular then show that it is regular. **6**

c) Let μ^* be a topologically regular outer measure on X then prove that each Borel set is μ^*- measurable. **4**

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT-704: Mathematical Methods – I (Old)

Time : 3 Hours Max. Marks : 80

N.B. *: i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Test for convergence the series **6**

$$\sum_{n=3}^{\infty} \frac{\sqrt{2n^2 - 5n + 1}}{4n^3 - 7n^2 + 2}$$

b) Find the interval of convergence of the power series **6**

$$\sum_{n=0}^{\infty} \frac{(X+2)^n}{\sqrt{n+1}}$$

c) Explain comparison test, integral test, ratio test for convergence of series of positive terms. **4**

2. a) Solve the boundary value problem

$Y_{tt}(X, t) = a^2 Y_{xx}(X, t)$, $0 < X < L$, $t > 0$

subject to conditions $Y(0, t) = 0$; $Y(1, t) = 0$, $Y_1(X, t) = 0$; $Y(X, 0) = f(X)$,

$0 \leq X \leq L$. **8**

b) Show that if f(X) has period P, then average value of f is the same over any interval of length P. **8**

3. a) Prove that, if two functions are expanded in Fourier series

$$f(X) = \frac{1}{2}a_0 + \sum_1^{\infty} a_n \cos nx + \sum_1^{\infty} b_n \sin nx$$

$$g(X) = \frac{1}{2}a'_0 + \sum_1^{\infty} a'_n \cos nx + \sum_1^{\infty} b'_n \sin nx$$

Then the average value of f (X) g (X) is $\frac{1}{4}a_0 a'_0 + \frac{1}{2}\sum_1^{\infty} a_n a'_n + \frac{1}{2}\sum_1^{\infty} b_n b'_n$. **8**

b) Find the sum of an infinite series f (X) = X on the interval $-1 < X < 1$ of period 2 by using Parseval's theorem. **8**

4. a) Given $f(X) = \begin{cases} 0, & 0 < X < l \\ 1, & l < X < 2l \end{cases}$

Expand f (X) in an exponential Fourier series of period 2*l*. **8**

b) Evaluate the following Γ (gamma) functions : **8**

i) $\Gamma\left(\frac{1}{2}\right)$ ii) Γ (5/3)

5. a) Prove that, $T = 4\sqrt{\frac{l}{2g}} \int_0^{\pi/2} \frac{d\theta}{\sqrt{\cos\theta}}$,

where T is period and *l* is length of simple pendulum. **8**

b) i) Define : error function

ii) Show that erf (–X) = – erf (X)

iii) Show that erf (∞) = 1. **8**

6. a) Use Laplace transform to solve the differential equation

$y''(t) + 4y'(t) + 13y(t) = 20e^{-t}$

subject to conditions y (0) = 1, $y'(0) = 3$. **6**

b) Find the value of $L^{-1}\left\{\log\left(\frac{s^2+1}{s(s+1)}\right)\right\}$ **6**

c) Obtain $L\{\sin^3 2t\}$. **4**

7. a) Find $P_0(X)$, $P_1(X)$, $P_2(X)$, $P_3(X)$ and $P_4(X)$ by using Rodrigue's formula. **10**

b) Show that $\int_{-1}^{1} [P_m(X)]^2 dX = \frac{2}{2m+1}$ with usual notations. **6**

8. a) Show that $\frac{d^s}{dX^s} H_n(X) = \frac{2^n n! H_{n-s}(X)}{(n-s)!}$ for X < n. **8**

b) Show that $L\{t^n f(t)\} = t^n \frac{d^n}{ds^n} f(s)$. **8**

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Seat No.	

M.A./M.Sc. (Semester – III) Examination, 2013
MATHEMATICS
MT – 705 : Graph Theory
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B.: *1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Prove that a graph is bipartite if it has no odd cycle. **6**

b) Prove that every graph with n vertices and k edges has at least n – k components. **4**

c) Prove that an edge is a cut edge if and only if it belongs to no cycle. **6**

2. a) Find the number of simple graphs with a vertex set X of size n. Draw all the non isomorphic simple graphs on a fixed set of four vertices. **5**

b) Prove that the Petersen graph has ten 6-cycles. **5**

c) Show that every set of six people contains at least three mutual acquaintances or three mutual strangers. **6**

3. a) Prove that for a connected nontrivial graph with exactly 2k odd vertices, the minimum number of trails that decompose it is max{k, 1}. **8**

b) Show that if G is a simple n-vertex graph with $\delta(G) \geq \frac{(n-1)}{2}$, then G is connected. **4**

c) Show that the number of vertices in a self-complementary graph is either 4k or 4k + 1, where k is a positive integer. **4**

4. a) Prove that if T is a tree k edges and G is a simple graph with $\delta(G) \geq k$, then T is a subgroup of G. **7**

b) Show that every graph has an even number of vertices of odd degree. **3**

c) Show that there exists a simple graph with 12 vertices and 28 edges such that the degree of each vertex is either 3 or 5. Draw this graph. **6**

P.T.O.

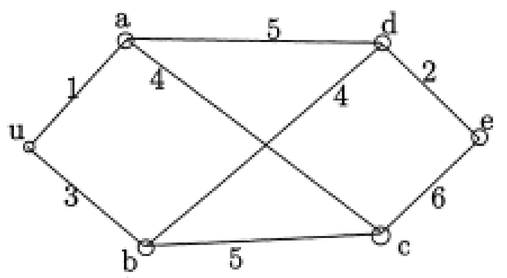
5. a) Prove that the center of a tree is a vertex or an edge. **7**

b) Let T be a tree with average degree a. Determine n(T) in terms of a. **3**

c) Prove that if G is a simple graph with diam $G \geq 3$, then diam $\overline{G} \leq 3$. **6**

6. a) Prove that if G is a bipartite graph, then the maximum size of a matching in G equals the minimum size of a vertex cover of G. **10**

b) Using Dijkstras algorithm find the shortest distance from u to every other vertex in the following graph. **6**



7. a) Prove that in a connected weighted graph G, Kruskal's Algorithm constructs a minimum-weight spanning tree. **6**

b) Determine whether the sequence (5, 5, 4, 3, 2, 2, 2, 1) is graphic. Provide a construction or a proof of impossibility. **4**

c) Show that the connectivity of the hypercube Q_k is k. **6**

8. a) Prove that every component of the symmetric difference of two matchings is a path or an even cycle. **5**

b) Define clique number and independence number of a graph G with an example.

Prove that for every graph $G, \chi(G) \geq \omega(G)$ and $\chi(G) \geq \dfrac{n(G)}{\alpha(G)}$, where $\omega(G)$ is the clique number of G and $\alpha(G)$ is the independence number of G. **5**

c) Prove that if G is a connected graph, then an edge cut F is a bond if and only if G – F has exactly two components. **6**

Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT-802 : Combinatorics (New)
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *1) Attempt **any five** questions.*
*2) Figures to the **right** indicate **full** marks.*

1. A) If n distinct objects are distributed randomly into n distinct boxes, what is probability that : **8**

   i) No box is empty ?

   ii) Exactly one box is empty ?

   iii) Exactly two boxes are empty ?

   B) How many arrangements of the 26 letters of the alphabet in which : **8**

   i) a occurs before b ?

   ii) a occurs before b and c occurs before d ?

   iii) the 5 vowels appear in alphabetical order ?

2. A) Suppose there are r_1 objects of type 1, r_2 objects of type 2, ..., r_m objects of type m and $r_1 + r_2 + ... + r_m = n$. Show that the number of arrangements of these n objects is $\dfrac{n!}{r_1!\, r_2! \ldots r_m!}$. **6**

   B) How many arrangements are possible with five letters chosen from MISSISSIPPI ? **6**

   C) How many arrangements are there of TINKERER with two but not three consecutive vowels ? **4**

3. A) How many ways are there to distribute 20 cents to n children and one parent if the parent receives either a nickel or a dime and

   i) The children receive any amounts ?

   ii) Each child receive at most 1₵ ? **6**

   B) How many 10 letter words are there in which each of the letters e,n,r,s occur

   i) at most once ? ii) at least once ? **6**

   C) How many arrangements are there of a,a,a,b,b,b,c,c,c without three consecutive letters the same ? **4**

4. A) Find a recurrence relation for the amount of money in a saving account after n years if the interest rate is 6 percent and $ 50 is added at the start of each year. **6**

   B) How many r-digit ternary (0, 1, 2) sequences are there with : **6**

   i) An even number of 0_s ?

   ii) An even number of 0_s and even number of 1s ?

   C) Find a recurrence relation for the number of ways to arrange cars in a row with n spaces if we can use Cadillacs or continentals or Fords. A Cadillacs or Continental requires two spaces, whereas a Ford require just one space. **4**

5. A) Find a recurrence relation for the ways to distribute n identical balls into K distinct boxes with between two and four balls in each box. Repeat the problem with balls of three colours. **6**

   B) Find ordinary generating function whose coefficient a_r equals 13.

   Hence evaluate sum

   13 + 13 + ... + 13. **4**

   C) Prove by combinatorial argument that $\binom{r}{r}+\binom{r+1}{r}+\ldots+\binom{n}{r}=\binom{n+1}{r+1}$

   Hence evaluate sum

   $1^2+2^2+3^2+\ldots+n^2$. **6**

6. A) How many ways are there to distribute 10 books to 10 children (one to a child) and then collect the books and redistribute them with each child getting a new one ? **6**

B) Solve the recurrence relation when $a_0 = 1$

i) $a_n = -na_{n-1} + n!$. **4**

C) How many ways are there to distribute 25 identical balls into seven distinct boxes if the first box can have no more than 10 balls but any amount can go into each of the other six boxes ? **6**

7. A) Using generating functions, solve the recurrence relation

$a_n = 3a_{n-1} - 2a_{n-2} + 2, \; a_0 = a_1 = 1.$ **6**

B) Solve the recurrence relation

$a_n = 2a_{n-1} + (-1)^n, \; a_0 = 2$. **4**

C) How many ways are there to arrange the letters in INTELLIGENT with at least two consecutive pairs of identical letters. **6**

8. A) State and prove Inclusion - Exclusion formula. **8**

B) Seven dwarfs $D_1, D_2, D_3, D_4, D_5, D_6, D_7$ each must be assigned to one of the seven jobs in a mine, $J_1, J_2, J_3, J_4, J_5, J_6, J_7$. D_1 cannot do jobs J_1 or J_3; D_2 cannot do J_1 or J_5 ; D_4 cannot do J_3 or J_6 ; D_5 cannot do J_2 or J_7; D_7 cannot do J_4. D_3 and D_6 can do all jobs. How many ways are there to assign the dwarfs to different jobs ? **8**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT 802 : Hydro Dynamics
(Old)

Time : 3 Hours Total Marks : 80

Instructions : *i) Attempt **any five** questions.*
*ii) Figures to the **right** indicate **full** marks.*

1. a) Derive relation for translation, rotational, and for deformation of a fluid element. **4**

   b) Derive conservation of mass by Lagrangian approach. **6**

   c) Velocity distribution of a certain flow is given by ; $V = i\,3xy^2 + j\,4x^2y + k\,5xyz$ obtain translation velocity, angular velocity and the rate of deformation of fluid elements. **6**

2. a) Define stream lines, path lines and streak lines. **8**

   b) A two dimensional flow towards a normal boundary is found to be characterized by a normal component of velocity that varies directly with distance from the boundary. Determine the stream function and the stream line (Assume velocity zero on the boundary). **8**

3. a) Derive Bernoulli's equation for an inviscid fluid. **8**

   b) Using Bernoulli's equation, show that velocity at the point of initial minimum cross section for an orifice is $v_0 = \sqrt{2}gh$. **8**

4. a) What is D'Alembert's Paradox ? Show that total force experienced by a body in the middle of the tube in the water is zero. **8**

   b) For a fluid with the given velocity held, identify whether fluid satisfy condition of incompressible property at (1, 1, 1). $V = 2x^2y\,i + 3y^2z\,j – 5z^2x\,k$. **8**

5. a) What is Kelvin's Circulation theorem ? Discuss in brief. **7**

   b) Derive the relation for a cylinder moving perpendicular to its axis in flow with circulation. **9**

6. a) What is source and sink ? Show that the relation for source and sink is $\varphi = m / 4\pi r$, $\varphi = - m / 4\pi r$ respectively where m is the strength. **8**

b) Derive general relation for family of stream lines for a simple source in uniform flow. **8**

7. a) Distinguish between viscous and non viscous fluid. **4**

b) Define viscosity and obtain its relation for a fluid obeying Newtonian Hypothesis (for 2-Dimensional). Define units of Viscosity. **6**

c) For a fluid having the velocity field (2-D) defined by $V = 4x^2y\, i + 3xy\, j$. Obtain the shearing stress acting at the point (1, 1) [given $\mu = 2.58P$]. **6**

8. Write short notes on **any two** of the following : **(8+8)**

a) Blasius theorem

b) Kutta - Juckowski theorem

c) Method of Images.

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT – 803 : Differentiable Manifolds
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. *: 1) Attempt* ***any five*** *questions.*
2) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Let M be a k – manifold in $\mathbb{R}^n$. If ∂M is nonempty, then prove that ∂M is a k – 1 manifold without boundary. **7**

   b) Let f and g are tensors on $\mathbb{R}^4$ given by
   $f(X, Y, Z) = 2x_1y_2z_2 - x_2y_3z_1$, $g = \phi_{2,1} - 5\phi_{3,1}$. Express $f \otimes g$ as a linear combination of elementary 5-tensors. **5**

   c) Give an example of a 1-mainifold in $\mathbb{R}^2$ which cannot be covered by a single coordinate patch. **4**

2. a) Define volume of a parametrized manifold and show that it is invariant under reparametrization. **8**

   b) Define an exact form and give an example. **4**

   c) Show that the sphere S^2 is a 2-manifold in $\mathbb{R}^3$. **4**

3. a) Define the differential operator d and for any k – form ω, show that $d(d\omega) = 0$. **8**

   b) If $\omega = xzdx + 2ydy + ye^z dz$ find $d\omega$. **4**

   c) Define a closed form and give an example. **4**

4. a) Let M be a compact k – manifold in $\mathbb{R}^n$ and $h : \mathbb{R}^n \to \mathbb{R}^n$ be an isometry. If N = h (M), then show that M and N have the same volume. **7**

   b) If $\omega = xz^2dx + 2ydy + ze^{-y}dz$ and $\eta = z \sin xdx + xzdy + ydz$, then find $d(\omega \wedge \eta)$. **5**

   c) Define 'Alternating Tensor' and give an example. **4**

P.T.O.

5. a) With usual notation, show that $\alpha^*(d\omega) = d(\alpha^*\omega)$. **8**

b) Let ω = xzdx + 2ydy – xdz, and α (u, v) = (uv, u^2,u + v). Find α^* ($d\omega$). **8**

6. a) What is the dimension of A^k (V), the space of alternating k – tensors on an n dimensional vector space V ? Justify. **7**

b) State Stokes's theorem. **5**

c) Define the term 'induced orientation'. **4**

7. a) If ω and η are k and l forms resectively, then prove that

$d(\omega \wedge \eta) = d\omega \wedge \eta + (-1)^k \omega \wedge d\eta$. **8**

b) Let $\alpha : (0, 1)^2 \to \mathbb{R}^3$ be given by α (u, v) = (u, v, $u^2 + v^2 + 1$). Let Y be the image set of α.

Evaluate $\int_Y x_2 dx_2 \wedge dx_3 + x_1 x_3 dx_1 \wedge dx_3$. **8**

8. a) Let M be a k – manifold in $\mathbb{R}^n$ and p$\in$ M. Define tangent space to M at p and show that the definition is independent of the choice of the coordinate patch at p. **8**

b) If G is a symmetric tensor, then show that AG = 0. **4**

c) Find centroid of the parametrized curve α(t) = (a cos t, a sin t), 0 < t < π. **4**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT-804 : Algebraic Topology
(2008 Pattern)

Time : 3 Hours Max. Marks : 80

N.B. : *i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Prove that there exists $f : B^2 \to S^{n-1}$ with $f.i = I$ if and only if the identity map $I : S^{n-1} \to S^{n-1}$ is homotopic to constant map, where $i : S^{n-1} \to B^n$ is the inclusion map and $\cdot$ denotes the scalar product. **6**

   b) Let $f : S^1 \to X$ be a continuous map. Show that f is null homotopic if and only if there is a continuous map $g : B^2 \to X$ with $f = g|S^1$. **5**

   c) Prove that the homotopy relation is an equivalence relation. **5**

2. a) Prove that if Y is contractible, then every continuous mapping $f : X \to Y$ is homotopic to a constant map. **6**

   b) Show that a retract of a Housdorff space is a closed subset. **5**

   c) Show that $\{0\} \cup \{1\}$ is not a retract of [0, 1]. **5**

3. a) Let f and g be paths such that $f * \overline{g}$ exists and is a closed path. Prove that $f * \overline{g}$ is homotopic to a null path if and only if f and g are equivalent. **6**

   b) Show that every path connected space is connected. Hence deduce that every convex subset of $\mathbb{R}^n$ is connected. **5**

   c) In $\mathbb{R}^2$, let $A = \{(x, y) : x = 0, -1 \leq y \leq 1\}$ and $B = \{(x, y) : 0 < x \leq 1, y = \cos \pi/x\}$. Show that $F = A \cup B$ is connected but not path connected. **5**

4. a) Let $x_0, x_1 \in X$. Prove that if there is a path from x_0 to x_1 then the groups $\pi_1(X, x_0)$ and $\pi_1(X, x_1)$ are isomorphic. **6**

   b) Define a contractible space and a simply connected space. Prove that a contractible space is simply connected but converse is not true. **5**

   c) If A is a strong deformation retract of X, then show that the inclusion map $i : A \to X$ induces an isomorphism $i^* : \pi_1(A, a) \to \pi_1(X, a)$ for any point $a \in A$. **5**

5. a) Prove that the fundamental group $\pi_1(S^1)$ of the circle S^1 is isomorphic to the additive group Z of integers. **8**

b) Prove that $\pi_1(\mathbb{R}^n, 0)$ is the singleton group. **4**

c) Show that the circle S^1 is not a retract of the disc B^2. **4**

6. a) Let X be a locally connected space and let $p : \tilde{X} \to X$ be a continuous map. Prove that p is a covering map if and only if for each component H of X, the map $p \mid p^{-1}(H) : p^{-1}(H) \to H$ is a covering map. **6**

b) Prove that a covering map is a local homeomorphism but the converse is not true. **5**

c) Let X be a G-space where G is a finite group. Show that the natural projection $\pi : X \to X / G$ is a closed mapping. **5**

7. a) Let $p : \tilde{X} \to X$ be a fibration with the unique path lifting. Suppose that f and g are paths in $\tilde{X}$ with f(0) = g(0) and pf is equivalent to pg. Prove that f is equivalent to g. **6**

b) Let $p : E \to B$ be a fibration. Prove that p(E) is a union of path components of B. **5**

c) Suppose $p : E \to B$ has unique path lifting. Prove that p has path lifting property for path connected spaces. **5**

8. a) Prove that the closed ball $B^n (n \geq 1)$ has the fixed point property. **7**

b) Suppose that K and L are complexes. Prove that if $|K \cap L| = |K| \cap |L|$ then $K \cup L$ is a complex. **5**

c) Prove that the surface of the sphere in $\mathbb{R}^3$ is a triangulable space. **4**

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Seat No.	

M.A./M.Sc. (Semester – IV) Examination, 2013
MATHEMATICS
MT-804 : Mathematical Methods – II (Old)
(2005 Pattern)

Time : 3 Hours   Max. Marks : 80

N.B. : *i) Attempt* ***any five*** *questions.*
ii) Figures to the ***right*** *indicate* ***full*** *marks.*

1. a) Define :
   i) Resolvent Kernel
   ii) Fredholm integral equation of the first kind. **4**

   b) Form the integral equation corresponding to the differential equation $y'' + y = 0$ with initial conditions $y(0) = y'(0) = 0$. **6**

   c) Prove that, the eigen values of a symmetric Kernel are real. **6**

2. a) Reduce the following boundary value problem into an integral equation

   $$\frac{d^2y}{dx^2} + xy = 1, \; y(0) = y(1) = 0.$$ **8**

   b) Show that the function $u(X) = \sin(\pi x/2)$ is a solution of the Fredholm integral equation $u(x) - \frac{\pi^2}{2}\int_0^1 k(x,t)\, u(t)dt = \frac{x}{2}$ **8**

3. a) Find the eigen values and eigen functions of the homogeneous integral equation

   $$\phi(X) = \lambda \int_{-1}^{1} (5Xt^3 + 4X^2t + 3Xt)\, \phi(t)dt$$ **8**

   b) Solve the homogeneous Fredholm integral equation of the second kind

   $$g(s) = \lambda \int_0^{2\pi} \sin(X + t)\, g(t)\, dt$$ **8**

4. a) Find the resolvent kernel of the Volterra integral equation with the kernel k(X, t) = 1. **6**

b) Find the Neumann series for the solution of the integral equation

$$y(X) = 1 + \int_0^X Xt\ Y(t)\, dt .$$ **10**

5. a) Prove that, if a kernel is symmetric then all its iterated kernels are also symmetric. **8**

b) Find the iterated kernels for the kernel K(X, t) = X – t ; a = 0, b = 1. **8**

6. a) Let $\psi_1(s), \psi_2(s), \ldots$ be a sequence of functions whose norms are all below a fixed bound M and for which the relation $\psi_n(s) - \lambda \int k(s,t)\ \psi_n(t)\ dt = 0$ holds in the sense of uniform convergence. Prove that the functions $\psi_n(s)$ form a smooth sequence of functions with finite a symptotic dimension. **8**

b) Find the plane curve of fixed perimeter and maximum area. **8**

7. a) Prove that, $\frac{d}{dx}\frac{\partial F}{\partial y'} - \frac{\partial F}{\partial y} = 0$

(Euler-Lagrange's equation) with usual notations. **8**

b) Show that the curve which extremizes the functional $I = \int_0^{\pi/4} (y''^2 - y^2 + X^2)\ dX$

under the conditions y(0) = 0, $y'(0) = 1$, $y\left(\frac{\pi}{4}\right) = y'\left(\frac{\pi}{4}\right) = 1/\sqrt{2}$ is y = sin x. **8**

8. a) State and prove Harr theorem. **8**

b) State and prove principle of Least Action. **8**
